The results obtained here – particularly those concerning the possibility of the formation of particle aggregates in a nonisothermal aerosol – may prove useful in performing specific calculations related to the dynamics of such systems. The results might also be used in the design and development of devices for removing aerosol particles from air.

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THERMOCAPILLARY CONVECTION WITH TEMPERATURE-DEPENDENT HEAT RELEASE AT AN INTERFACE

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It is known that thermocapillary instability in a system with an interface can be either monotonic or oscillatory in character [1-4]. The stability of the equilibrium state of the system is significantly influenced by presence of heat sources and sinks on the interface due to chemical reaction, evaporation, absorption of radiation, etc. The study [5] solved the problem of stability of equilibrium in a two-layer system against mononotonic perturbations under conditions of surface heat release. Nepomnyashchii and Simanovskii [6] examined the stability of a two-layer system against monotonic and oscillatory perturbations in the presence of temperature-independent heat release at the interface.

In the present investigation, we solve the same problem with allowance for the temperature dependence of surface heat release. It is shown that in certain cases this dependence can lead to expansion of the region associated with oscillatory instability.

1. Let the space between two solid horizontal plates $y = a_1$ and $y = -a_2$, kept at constant temperatures T_1 and T_2, be filled by two layers of viscous immiscible fluids. The x-axis is directed horizontally, while the y axis is directed vertically upward. We assume that thermocapillary convection occurs in the presence of gravity, which in turn allows us to consider the interface to be planar and nondeformable ($y = 0$). Despite this, the effect of buoyancy on convection is assumed to be negligible compared to the thermocapillary effect – as is seen for thin films of liquid. The absolute and kinematic viscosities, thermal conductivities, and diffusivities are equal to η_m, ν_m, κ_m, χ_m ($m = 1$ for the top fluid and $m = 2$ for the bottom fluid). Surface tension is linearly dependent on temperature: $\sigma = \sigma_0 - \alpha T$.

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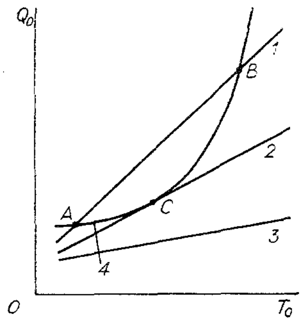

Fig. 1

We will assume that the quantity of heat Q_Γ, determined by the temperature of the interface T_0, is released at the boundary between the media (such heat release occurs, for example, in the case of a heterogeneous chemical reaction). Figure 1 (line 4) shows a typical dependence of heat release on temperature (with a heterogeneous chemical reaction). For conditions of mechanical equilibrium, Q_0 and the vertical temperature gradients A_m ($m = 1, 2$) are calculated from the heat-balance condition $-\kappa_1 A_1 + \kappa_2 A_2 = Q_0$ (the quantity of heat generated at the interface between the fluids must be equal to the quantity of heat removed to the solid boundaries of the system) and the relation $A_1 a_1 + A_2 a_2 = -s\theta$ ($s = 1$ with heating from below, $s = -1$ with heating from above); $\theta = |T_1 - T_2|$. We find that $A_1 = -(s\theta\kappa_2 + Q_0 a_2)/(a_1\kappa_2 + a_2\kappa_1)$, $A_2 = -(s\theta\kappa_1 - Q_0 a_1)/(a_1\kappa_2 + a_2\kappa_1$. The equilibrium values of the temperature of the boundary T_0 and heat release Q_0 are determined from the system of equations

$$T_0 = \frac{T_1 \varkappa_1 a_2 + T_2 \varkappa_2 a_1 + Q_0 a_1 a_2}{a_1 \varkappa_2 + a_2 \varkappa_1}; \tag{1.1}$$

$$Q_0 = Q_\Gamma(T_0). \tag{1.2}$$

Figure 1 shows different possibilities for the mutual location of the graphs of Eq. (1.1) (lines 1-3) and (1.2) (line 4). It is clear that the system can have two solutions (if the thermal conductivity of the medium is great enough), one solution (in the critical case), or no solutions (if not enough heat is removed). We will henceforth assume that there is but one solution to system (1.1)-(1.2).

We introduce the following notation: $\eta = \eta_1/\eta_2$, $\nu = \nu_1/\nu_2$, $\kappa = \kappa_1/\kappa_2$, $\chi = \chi_1/\chi_2$, $a = a_2/a_1$. We choose a_1, a_1^2/ν_1, ν_1, and θ as the units of length, time, the stream function, and temperature, respectively. Under equilibrium conditions, the dimensionless temperature gradient is equal to $A_1 = -(s + Qa\kappa)/(1 + \kappa a)$ in the top fluid and $A_2 = -\kappa(s - Q)/(1 + \kappa a)$ in the bottom fluid, where $Q = Q_0 a_1/\theta\kappa_1$.

We impose perturbations of the stream function ψ_m' and temperature T_m' on the equilibrium state:

$$(\psi_1', T_1', \psi_2', T_2') = (\psi_1(y), T_1(y), \psi_2(y), T_2(y)) \exp[ikx - (\lambda + i\omega)t]$$

(k is the wave number; $\lambda + i\omega$ is the complex decrement).

The linearized equations for the perturbations of the stream function and temperature have the form [4]

$$\begin{gathered}(\lambda + i\omega) D\psi_m = -d_m D^2 \psi_m, \\ -(\lambda + i\omega) T_m - ik\psi_m A_m = (e_m/\mathrm{Pr}) D T_m,\end{gathered} \tag{1.3}$$

where $D = d^2/dy^2 - k^2$: $b_1 = e_1 = 1$; $d_2 = 1/\nu$; $e_2 = 1/\chi$; $\mathrm{Pr} = \nu_1/\chi_1$ is the Prandtl number.

Using a prime to denote differentiation with respect to y, we write the conditions for the solid boundaries

$$y = 1:\ \psi_1 = \psi_1' = T_1 = 0, \quad y = -a:\ \psi_2 = \psi_2' = T_2 = 0 \tag{1.4}$$

and for the interface

$$y=0:\ \psi_1=\psi_2=0,\ \psi_1'=\psi_2',\ T_1=T_2,\ \varkappa T_1'=T_2'-Q_T T_1,$$
$$\eta\psi_1''-ik\,\mathrm{Mr}\,T_1=\psi_2'',\ \mathrm{Mr}=\alpha\theta a_1/(\eta_2\nu_1),\ Q_T=(dQ_\Gamma/dT_0)\,a_1\varkappa_2^{-1}. \tag{1.5}$$

The boundary of the stable state is determined by the condition $\lambda = 0$. In the approximation being examined, the terms $ikGb_mT_m$ — which describe buoyancy (where $G = g\beta_1\theta a_1^3/\nu_1^2$ is the Grashof number, $b_1 = 1$, $b_2 = \beta_2/\beta_1$, β_m is the coefficient of thermal expansion of the m-th fluid) — are dropped from the equations for the perturbations of the stream function [7]. If the densities and coefficients of thermal expansion of the two media are close enough to one another, then the validity of the given approximation can be evaluated by using the criterion established for convection in a one-layer system [8]:

$$a_m \ll a_c,\ a_c \sim \min_m\,(\alpha/g\rho_m\beta_m)^{1/2}. \tag{1.6}$$

It is convenient to introduce the parameter $\mathrm{Mr}_Q = \mathrm{Mr}Q = \alpha Q_0 a_1^2/(\eta_2\nu_1\kappa_1)$ to analyze the effect of surface heat release on the onset of thermocapillary instability. In contrast to Q, this parameter is independent of θ and remains constant with a change in the difference in temperature between the top and bottom boundaries of the system. Different values of Mr_Q correspond to different rates of heat release at the interface.

2. Boundary-value problem (1.1)-(1.5) for monotonic instability ($\lambda = \omega = 0$) can be solved analytically [5]. The expression for the critical value of Mr_Q has the form

$$\mathrm{Mr}_Q=\frac{s\,\mathrm{Mr}\,(\chi C_2-C_1)-8\,(1+\varkappa a)\,(\varkappa\,\mathrm{Pr})^{-1}\,k\,[k\,(\varkappa D_1+D_2)-Q_T]\,(\eta B_1+B_2)}{\chi C_2+\varkappa a C_1}. \tag{2.1}$$

Here

$$B_1=(s_1c_1-k)/(s_1^2-k^2);\ B_2=(s_2c_2-ka)/(s_2^2-k^2a^2);$$
$$C_1=(s_1^3-k^3c_1)/[s_1(s_1^2-k^2)];\ C_2=(s_2^3-k^3a^3c_2)/(s_2^2-k^2a^2);$$
$$D_1=c_1/s_1;\ D_2=c_2/s_2;\ s_1=\mathrm{sh}\,k;\ s_2=\mathrm{sh}\,ka;\ c_1=\mathrm{ch}\,k;\ c_2=\mathrm{ch}\,ka.$$

The equilibrium is stable at $\mathrm{Mr}_Q > \mathrm{Mr}_{Q*}$, where $\mathrm{Mr}_{Q*} = \max \mathrm{Mr}_Q(k)$. This means that heat release has a stabilizing effect on the equilibrium state, while heat absorption has a destabilizing effect [5, 6]. It is evident from Eq. (2.1) that an increase in Q_T leads to an increase in $\mathrm{Mr}_Q(k)$ (destabilization) and that a decrease in Q_T leads to a decrease in $\mathrm{Mr}_Q(k)$ (stabilization).

In fact, the formulation of the problem of the onset of thermocapillary convection has significance only for the region

$$Q_T < Q_{T*} = \varkappa + 1/a, \tag{2.2}$$

where the neutral curve (2.1) has an extremum. It is not hard to see that condition (2.2) is satisfied for equilibrium — which corresponds to point A in Fig. 1 — and is not satisfied for point B. The equality $Q_T = Q_{T*}$ is valid for point C. In the region $Q_T > Q_{T*}$ for any value of Mr, the equilibrium state is unstable against spatially uniform ($k = 0$) and long-wave (small k) perturbations due to the phenomenon of thermal shock [5].

Let us examine the special case $\chi = 1$, $a = 1$. In the absence of heat sources and sinks, monotonic instability is not seen for this case. In the presence of heat absorption, the monotonic neutral curve has the form

$$\mathrm{Mr}_Q(k)=-\frac{8\,(1+\varkappa)\,(1+\eta)\,k\,[k\,(\varkappa+1)\,c_1-Q_Ts_1]\,(s_1c_1-k)}{\varkappa\,\mathrm{Pr}\,(s_1^3-k^3c_1)} \tag{2.3}$$

and is independent of the parameter Mr.

To obtain the boundaries of the region associated with oscillatory instability, we need to solve the problem numerically. We will examine a system with the parameters $\eta = \nu = 0.5$, $\kappa = \chi = \mathrm{Pr} = a = 1$. We will limit ourselves to the case of heating from below. Monotonic instability is realized at $\mathrm{Mr}_Q < \mathrm{Mr}_{Q*} < 0$, where Mr_{Q*} is the extremum of Eq. (2.3). At

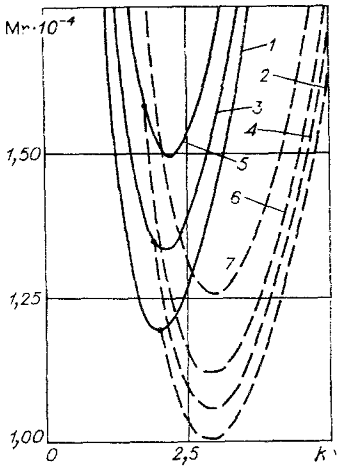

Fig. 2

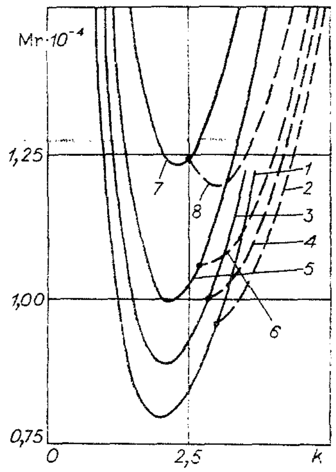

Fig. 3

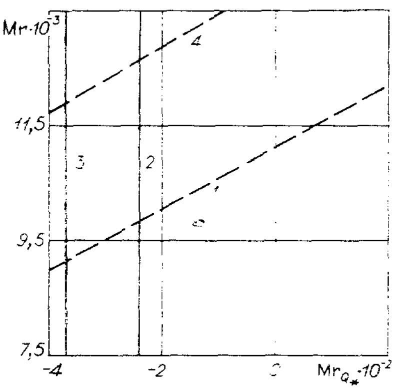

Fig. 4

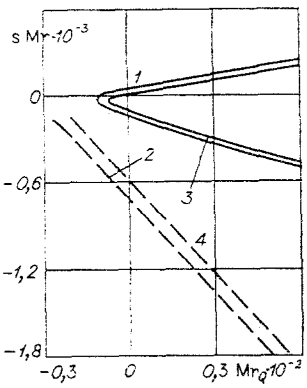

Fig. 5

$Mr_Q > Mr_{Q*}$, oscillatory instability is the only possible mechanism of destabilization of the equilibrium state. Figure 2 shows the neutral curves for $Q = -0.02$; $Q_T = 0$ (lines 1, 2), -0.5 (3, 4), -1.1 (5, 6), -2.4 (line 7). Here and below, the monotonic neutral curves are shown by solid lines, while the oscillatory curves are shown by dashed lines. The neutral curves in Fig. 3 were constructed for $Q = -0.03$; $Q_T = 0$ (lines 1, 2), -0.5 (3, 4), -1.1 (5, 6), -2.4 (7, 8). It is evident from Figs. 2 and 3 that both the monotonic and oscillatory modes of instability become more stable with an increase in the parameter $|Q_T|$ ($Q_T < 0$). Figure 4 shows the boundaries of the stability region obtained by determining the extrema of the neutral curves for monotonic and oscillatory perturbations [$Q_T = 0$ (lines 1, 2), -2.4 (3, 4)]. With an increase in $|Q_T|$, the region of oscillatory instability expands due to the reduction in the value of Mr_{Q*} corresponding to the threshold of monotonic instability.

Now let us examine a system of real fluids composed of transformer oil and formic acid. The system is characterized by the following set of physical parameters: $\eta_1 = 0.0198$ N·sec/m², $\nu_1 = 0.225 \cdot 10^{-4}$ m²/sec, $\kappa_1 = 0.111$ W/(m·K), $\chi_1 = 0.736 \cdot 10^{-7}$ m²/sec, $\eta_2 = 0.178 \cdot 10^{-2}$ N·sec/m², $\nu_2 = 0.146 \cdot 10^{-4}$ m²/sec, $\kappa_2 = 0.271$ W/(m·K), $\chi_2 = 1.03 \cdot 10^{-7}$ m²/sec, $\beta_1 = 0.692 \cdot 10^{-3}$ 1/K, $\beta_2 = 1.03 \cdot 10^{-3}$ 1/K. The dimensionless parameters for this system are equal to: $\eta = 11.1$, $\nu = 15.4$, $\kappa = 0.41$, $\chi = 0.714$, $Pr = 306$. It should be noted that, according to (1.6), the thicknesses of the layers for which the given approximation is invalid are $a_c \sim 4$ mm [we assume that $\alpha \sim 0.1 \cdot 10^{-3}$ N/(m·K)]. Let us discuss the case $a = 1.667$. The thermal-shock threshold for this system is reached at $Q_T = Q_{T*} = 1.01$. Figure 5 shows the boundaries of the regions in which the system is stable against monotonic and oscillatory perturbations for $Q_T = 0$ (lines 1, 2) and 1.01 (3, 4); the corresponding type of instability is realized in the region to the left of the boundaries shown in the figure. It is evident that an increase in Q_T leads to destabilization of equilibrium in relation to both modes of instability, although monotonic instability remains the greater danger. At $Q_T > Q_{T*}$, the equilibrium state is unstable relative to monotonically increasing long-wave perturbations for any sMr, Mr_Q. Figure 6 shows the change in the form of the neutral curves at $Q = 0.03$ [$Q_T = 0$ (lines

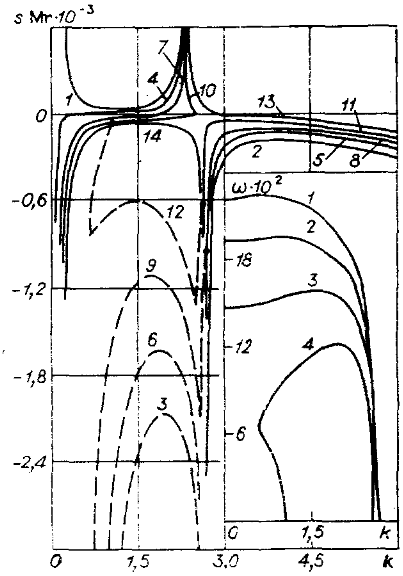

Fig. 6

1-3), 1.1 (4-6), 2.4 (7-9), 3.6 (10-12), 4.8 (13, 14)] and the dependence of the frequency of oscillation on the wave number at $Q_T = 0$; 1.1; 2.4; 3.6 (lines 1-4) in the thermal-shock region. Long-wave disturbances ($k \to 0$) always increase in this region. For finite k and sufficiently small Q_T, the region in which the perturbations decay is complicated in form. For sufficiently large Q_T, the oscillatory mode of instability disappears and the region of decay of the perturbations takes the form $k > k_*$; with an increase in Mr, the boundary wave number k_* decreases at $s > 0$ (heating from below) and increases at $s < 0$ (heating from above).

Thus, the dependence of surface heat release on temperature has a significant effect on monotonic and oscillatory modes of thermocapillary instability. Here, an increase in heat release with an increase in temperature ($Q_T > 0$) has a destabilizing effect, while a decrease ($Q_T < 0$) has a stabilizing effect.

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